THE MULTIPLIER TIME PERIOD AND THE INCOME VELOCITY OF ACTIVE MONEY

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I

Some economists, notably Mr. Fritz Machlup, Mr. Richard M. Goodwin, Mr. James W. Angell, and Mrs. Vera Lutz, have maintained that the multiplier time period is determined by the magnitude of the income velocity of active money.[1] Others, especially Mr. Paul A. Samuelson and Mr. Gardner Ackley, have argued that such is not the case.[2]

The most important source for the dispute has been that the multiplier has been differently defined. For purposes of this paper there have been two concepts of the multiplier. Mr. Angell and Mrs. Lutz have concerned themselves primarily with a multiplier defined in terms of monetary flows.[3] They imply the existence of a corresponding multiplier accounting for the flow of goods and services. Those with a different conception of the multiplier are in two camps. Mr. Machlup and Mr. Goodwin have defined the multiplier in terms of the flow of goods and services, but the goods and services represented are those whose movement has been made possible by the flow of money originating at one point.[4] Another group has followed the lead of John M. Keynes, who defined the multiplier in terms of the flow of goods and services but who did not specify whether the money moving this flow is to be injected at one or more

[1] James W. Angell, *Investment and Business Cycles* (New York: McGraw-Hill Book Company, 1941), pp. 187–210; Richard M. Goodwin, "The Multiplier," *The New Economics* (New York: Alfred A. Knopf, 1947), ed. by S. E. Harris, pp. 487–493; Vera Lutz, "Multiplier and Velocity Analysis: A Marriage," *Economica*, N. S., 1955, XXII, pp. 29–44; Fritz Machlup, "Period Analysis and Multiplier Theory," *The Quarterly Journal of Economics*, 1939, LIV, pp. 1–27, reprinted in *Readings in Business Cycle Theory* (Philadelphia: The Blakiston Company, 1944), pp. 203–234. Subsequent references to Machlup's article will be to the reprint.

[2] Gardner Ackley, "The Multiplier Time Period: Money, Inventories, and Flexibility," *The American Economic Review*, 1951, XLI, pp. 350–355; Paul A. Samuelson, "Fiscal Policy and Income Determination," *The Quarterly Journal of Economics*, 1941–1942, LVI, p. 602.

[3] Mr. Angell's thinking is oriented toward explaining income generation in terms of monetary flows. It is not surprising that as a refinement of his basic analysis, he translates the approach to income generation by means of the quantity of active money and its income velocity into multiplier terms. It is not misleading to say that he defines a multiplier in terms of monetary flows. His multiplier is the reciprocal of the marginal propensity to hoard, and hoarding is generally defined in monetary terms. Parts IV and V of the Lutz article listed in fn. 1 are a defense and a refinement of Angell's analysis.

[4] By defining the multiplier in terms of the marginal propensity to consume, Mr. Machlup follows a practice favored by those who think of the multiplier in terms of the flow of goods and services. He makes it clear that he appeals to the concept of the income velocity of active money only in an effort to determine the time dimension of the multiplier. (See Machlup, *op. cit.*, especially p. 220, fn. 1.) In his model Mr. Machlup will not permit the realization of a term of the multiplier until active money has completed a cycle from the consumers

points.[5] The dispute has been complicated by the failure of the parties concerned to make clear that they are speaking of multipliers differently conceived. In fact, some of them have implied that their opponents really do have the same multiplier in mind as they.[6]

This paper is not an attempt to evaluate the comparative merits of the approach to income generation by use of the quantity of active money and its income velocity. It does try to answer two questions: (1) Can the time period of a multiplier defined in terms of monetary flows be determined by use of the income velocity of active money? (2) Can the time period of a multiplier defined in terms of the flow of goods and services be ascertained by use of the income velocity of active money? Problems introduced by changes in the price level will not be dealt with, and a closed economy will be assumed.

II

This section deals with the relationship between the income velocity of active money and the multiplier concept defined in terms of monetary flows. As mentioned beforehand, this type of multiplier has been described and used by Mr. Angell and Mrs. Lutz. If s denotes the marginal propensity to spend for consumption and investment, the marginal propensity to hoard is denoted by $1 - s$. Let $1 - s$ be indicated by h. The hoarding denoted by h need not leave a hoard in the hands of individuals or non-banking business firms but may simply re-

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through retailers, wholesalers, manufacturers, and back to the consumers again. (*Ibid.*, pp. 216–226).

That Mr. Goodwin wishes to think primarily in terms of the flow of goods and services is indicated by the following: (1) He defines the multiplier in terms of the flow of goods and services—a procedure not favored by those strongly oriented toward monetary flows (*op. cit.*, p. 483); (2) he speaks of the multiplier in terms of consumption and investment (*ibid.*, p. 485); (3) he regards himself as borrowing the concept of income velocity from monetary theory (*ibid.*, pp. 487–488), whereas Angell borrows the multiplier concept and applies it to monetary flows.

[5] Keynes had a strong dislike for the concept of income velocity and defined the multiplier in terms of flows of investment and consumption. (See his *The General Theory of Employment, Interest and Money* (New York: Harcourt, Brace and Company, 1936), especially pp. 115, 194–195, 299.)

Mr. Ackley is clearly speaking of the multiplier in terms of the flow of goods and services. (See especially pp. 352–353 of the article cited in fn. 2.) Section III of this paper is a refinement of Ackley's contribution.

[6] Mr. Angell seems to argue that, had Keynes' thinking advanced, he would have stated the multiplier in terms of monetary flows. It is difficult to nail down this charge by citation to specific pages in Angell's *Investment and Business Cycles* because this position appears to underlie Angell's thinking throughout Chapters IX, X, and XI. Angell implies again and again that his thinking is simply the maturation of that of Keynes.

Mrs. Lutz clearly recognizes that there are multipliers defined in terms of the flow of goods and services and those set forth in terms of the flow of money. (See pp. 30-31 of her article cited in fn. 1.) She does not make it clear, however, that the time period of velocity analysis does not apply to the two types of multipliers in the same way.

Mr. Ackley, who thinks of the multiplier from the standpoint of the flow of goods and services, does not make it clear that some of his opponents are speaking with monetary flows primarily in mind.

duce the amount of money held by the public as in the retirement of debts to the money-creating banks or may result in the passage of money into financial circulation.[7]

A multiplier k_1 is defined by $1/h$, which is equal to $1/1 - s$. Also,

$$k_1 = 1 + s + s^2 + s^3 + s^4 + s^5 + s^6 + \ldots + s^n.$$

This multiplier shows that one monetary unit injected as active money results in the flow of spending indicated by this progression. If the marginal propensity to spend is .9, the sum of the terms of the series will approach 10 as a limit. The injected monetary unit will eventually result in the clearing of accounts valued at 10 times its worth.

The marginal propensity to spend is so defined that this progression also represents a flow of goods and services. Though the orientation is from the side of the flow of money, the flow of active money is conceived to be opposed by an equal flow of goods and services.

Since k_1 is defined in terms of the flow of active money, the rate at which successive terms of k_1 are realized is determined by the rate of flow of the active money involved.[8] A measure of the average rate of this flow is the ratio of the multiplier limit, which in the case described in the preceding paragraph is 10, divided by the average amount of active money associated with the realization of k_1. If successive terms of k_1 are realized in periods of equal length, the amount of active money involved on the average is simply $(1 + s + s^2 + s^3 + \ldots + s^n)/N$, where N represents the number of periods. For example, when k_1 is 10, the average amount of active money needed for the realization of the first three terms is $(1 + .9 + .81)/3$, or .90 1/3. During this period required for the attainment of the first three terms, the velocity ratio, which indicates the income velocity, is 2.71/.90 1/3, or 3, which indicates that one term will be realized in 1/3 the time required to obtain three.[9] If the rate of the flow of money varies from one term to the next, appropriate weighting must, of course, be used in computing the average.

If the income velocity of active money, or more exactly that of an increment of active money, can be used to determine the time span between the realization of one term of k_1 and the succeeding term, can the period of time required for the realization of the entire multiplier progression be determined by use of this measure of velocity? Obviously, when the marginal propensity to spend is unity, the period required for the attainment of k_1 cannot be determined by use of this velocity ratio, for this multiplier is infinite. Theoretically, even when s is below unity, k_1 is still the summation of an infinite series. From this standpoint, the time period necessary for the realization of k_1 is without end. The

[7] See James W. Angell, *op. cit.*, pp. 35, 166.

[8] Active money includes monetary balances for which individuals and institutions have definite plans to spend in some way other than in financial circulation.

[9] If active money is indicated by M, its income velocity by C, and the national income by Y, then the average income velocity of the entire stock of active money is denoted as follows: $C = Y/M$. The time period required for an average unit of active money to have an income velocity of 1 is the fraction of the time equal to the reciprocal of C. This time period is sometimes called an income period.

general conclusion is that the income velocity determines only the time period required for the accomplishment of a truncated k_1.

What has been gained by the statement of the multiplier in terms of monetary flows? The effect on the quantity of active money and on income of a continued injection of a given size per time period can be found by the same summation procedure as is commonly used in connection with multipliers.[10] When this process is employed in connection with k_1, the amount of active money in circulation and the amount of income generated in a given time period are presented as functions of income velocity and of marginal propensities to spend and to hoard. In this way the approach to income generation by use of the concepts of active money and its income velocity is presented in terms of the multiplier theory. The introduction of the multiplier concept does not amount to a fundamental change in this approach, which has the same virtues and defects that it formerly had. This approach will account for all flows of goods and services involving monetary flows, but the investigation of the relative merits of this type of analysis of income generation is not the purpose of this paper.

III

Following the example set by John M. Keynes in his *General Theory*, some economists define the multiplier in terms of the flow of goods and services. Generally these economists define the multiplier as the reciprocal of the marginal propensity to save, though it is entirely possible to state the multiplier in terms of monetary flows and set it forth in terms of the marginal propensity to save. If k_2 denotes the multiplier in terms of the flow of goods and services, if s indicates the marginal propensity to save, and c signifies the marginal propensity to consume,

$$k_2 = 1/s = 1/1 - c = 1 + c + c^2 + c^3 + c^4 + \ldots + c^n.$$

If the economists represented by Mr. Angell and Mrs. Lutz borrow the multiplier concept for the purpose of refining their approach to income generation by way of monetary flows, some economists like Mr. Machlup and Mr. Goodwin start out with the multiplier conceived in terms of the flow of goods and services and call in the income velocity of active money in an effort to measure the time dimension of this flow. Mr. Machlup is very explicit in his exposition.[11]

The attainment of the initiating term (1) of k_2 is accompanied by the injection of an equivalent amount of active money. The next term cannot, according to Machlup and Goodwin's viewpoint, be had until the unsaved part (c) of the initial injection of active money has travelled from the consumer through the retailers, the wholesalers, the manufacturers, and so on and back again to the consumers. The third term (c^2) cannot be realized until active money in the amount of c^2, derived from the initial injection of unity, has described a similar

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[10] For early demonstrations of this process, see Fritz Machlup, *op. cit.*, p. 224, and Paul A. Samuelson, "Interactions between the Multiplier Analysis and the Principle of Acceleration," *The Review of Economic Statistics*, reprinted in *Readings in Business Cycle Theory* (Philadelphia: The Blakiston Company, 1944), p. 266.

[11] Fritz Machlup, *op. cit.*, pp. 216–226.

cycle. For this model, it is correctly concluded that the time span between the accomplishment of one term and that of its successor is rigidly set by the income velocity of active money associated with this multiplier.

Machlup's model, which is followed also by Goodwin, is needlessly rigid. Other economists define the multiplier in the manner of k_2 and insist that the realization of a truncated k_2 does not depend solely upon the magnitude of the income velocity of active money. Monetary theorists long ago provided them with the foundation for their position when they announced that the flow of goods and services depends on the quantity of active money and its income velocity and not on one alone.

Is it, for example, necessary that k_2 be attained at a faster rate in an economy in which the income velocity of active money is high than in one in which the income velocity of active money is lower? That such is not the case can be easily demonstrated. Let it be assumed that in one economy six terms of k_2 are realized in one month and the flow of money is such that all accounts are cleared during the final week of this period. Now let it be imagined that there is another economy in which six terms of k_2 are attained in one month but in which accounts are cleared during the final week of each second month. For a given income twice as much money will be needed in the second as in the first, but the income velocity of active money in the latter model will be only one-half that in the first. Multipliers defined in terms of the flow of goods and services are attained as quickly in one as in the other. What is accomplished in the first by the higher income velocity is brought about by a larger amount of active money in the second.

In a given economy in which the arrangements for the clearing of accounts are such that the income velocity of active money is constant, the speed with which a truncated k_2 is attained need not be limited at all by the fixity of this income velocity. Log jams caused by the constancy of the income velocity of active money can be broken by the injection of more active money. Suppose that the income velocity of active money is such that, without additions to the quantity of active money, one year is required for the realization of five terms of a representative k_2. Thus, in a year the following are had: $1 + c + c^2 + c^3 + c^4$. If monetary factors alone are limiting and if the quantity of active money is readily augmentable, all five terms or even more can be got as quickly as the people wish by simultaneously injecting active money at numerous points. Those who wish to buy the initiating amount of unity (the first term) are to be provided through the money-creating institutions with one unit of money. At the same time the purchasers of c of consumer goods can be furnished this amount of money, the buyers of c^2 consumer goods can be supplied with a sum of money equal to c^2, and so on. All purchases can be made in short order. A representative k_2 can be brought to fruition by initiating one or many k_1's.

The general point is that the flow of goods and services is made possible in a money-using economy by the quantity of active money and its income velocity. If the quantity of active money or its income velocity can be increased to accommodate an increasing flow of goods and services, this flow need not be limited by monetary factors.

Those who have disagreed on the question of the possibility of using the income velocity of active money to determine the time period of the multiplier have all been correct. To make this statement is to declare that in the final analysis there has been no disagreement or that there should not have been. Those who define the multiplier in terms of monetary flows are right when they maintain that the time period required for the attainment of a truncated multiplier of their type is determined by the income velocity of active money alone. Those who define the multiplier in terms of the flow of goods and services are also correct when they argue that the time required for the accomplishment of a truncated multiplier of their type need not depend on the income velocity of active money alone but may depend on both the quantity of active money and its income velocity. Each side in the dispute has nothing to lose by admitting that the flow of goods and services need not be limited by monetary factors.